Comments on Lepore and Ludwig "Conceptions of Logical Form"

Robert May
University of California, Irvine

Over the years, I've been asked many times what "logical form" is, as applied to natural language. This is a natural enough question to address to me; after all, I've written a book titled *Logical Form*, and I've been asked to write any number of papers on the topic. This question, it seems to me, is certainly a "big" question, and big questions deserve big answers. I must admit, however, to being somewhat baffled as to how to do this satisfactorily, since big answers to big questions unfortunately tend to the trivial. With a nod to Wittgenstein, logical form has always seemed to me to be something that you know it when you see it; it is clear enough when it pops up, but one is hard pressed to say just what it *is*, to *define* it. This is so even though the meanings of the words "logical" and "form" seem straightforward enough; what I find puzzling is how the first word is supposed to modify the second. What is it that makes a form *logical*, as opposed to something else that is *not* logical? This, it seems to me, is a very hard question to answer indeed, for if we cannot contrast logical form with some other type of form, then every form (or no form) is a logical form, and we have arrived at the triviality previously mentioned.

On classical uses of the term logical form, as we find it in the grand tradition stemming from Frege, Russell, Wittgenstein, Tarski, Carnap, Quine, etc, there is a type of form which is distinct from logical form, namely grammatical form. In the classic example of Russell's Theory of Descriptions, the simple subject-predicate grammatical form of "The present King of France is bald" cannot be its logical form, for if it were incorrect inferences would follow. Rather, through the method of contextual definition, the grammatical form can be "translated" into another form, its *logical form*, (more precisely into a general proposition with the

proper logical form), from which the correct inferences follow. On the usual run of things on this view, the translation is into some other logically precise language, since, it is argued, there are no systematic ways of paraphrasing within the natural language itself which will systematically reveal logical form.

This answer is certainly not trivial, and retains its popularity in many quarters, oftentimes in technically very sophisticated forms; it is not, however, a view to which Ernie Lepore and Kirk Ludwig subscribe, as you have heard. Rather, they are pushing a view of logical form rather more integral to the interpretation of natural languages. In a nutshell, their answer to the question of what is logical form is that it is the structure relevant to the proofs of T-sentences in an interpretative truth theory; they speak of logical form as a *property* of a sentence that is "revealed" by such proofs. I think I agree with what they say, or at least I want to, since I too like interpretative truth theories as semantic theories of natural language, and I do think that whatever logical form is, it is intimately related to whatever form it is that such truth theories interpret. My hesitation arises because their view, unlike the one they reject, apparently *is* trivial, at least to the extent that the theory is compositional, (at least in one sense of the term). Let me explain.

An interpretive truth theory will contain axioms roughly of the form "x is an interpretation of α iff . . .", where α is some syntactic structure, be it a word, a phrase or a clause, where, depending upon which it is, the ellipsis will be filled in various ways. In general, if α is a complex syntactic structure, made up of the immediate constituents ψ and φ, then x is an interpretation of α iff for some y, y is an interpretation of ψ and y is an the interpretation of φ, and if α is a word, then the interpretation of α is whatever it is specified to be in the lexicon (for instance, if α is a name, then its interpretation is its reference). It should be clear that an interpretative truth theory whose axioms conform with these conditions will be compositional in the usual sense that the way that a sentence's T-sentence is constructed (proved) will directly mirror its immediate constituent analysis, and that the complexity of the proof will be computable from the complexity of the interpreted structure. It should also be clear that for any language L interpreted in this way, that grammatical form - that is, the form specified by the syntax of the language - will be logical form, on the Lepore/Ludwig

conception, for clearly this is the structure which is relevant to the proofs of the T-sentences for the sentences of *L*. This structure could then be represented through any standard syntactic notation (e.g. tree structures). Thus, insofar as there is a strictly compositional interpretation of *L*, we have a trivial characterization of logical form, for logical form is simply the syntactic structure which is interpreted by the truth theory.

Lepore and Ludwig, recall, hold that logical form is a "property" of a sentence; "This property," they say, "is determined if we can characterize when two sentences . . . share logical form." (p. 9) Sentences *sharing a logical form* or *having the same logical form*, is then defined by their having isomorphic canonical proofs of their T-sentences; that is, proofs which are the same save for the contributions of particular choice of lexical items to the proofs of the T-sentences. For example, "John left and "Mary came" have distinct T-sentences; nevertheless, they have the same logical form, since "John" and "Mary" have the same type of axiom - "name axioms" - and so do "left" and "came" - "one-place predicate axioms." (Cf. clause (c) of the definition of corresponding proof.) Since the particular choice of words is abstracted away from, we can think of this as a "coarse-grained" conception of logical form, in which logical forms are types of sentence types, in contrast to "fine-grained" conceptions we find in the literature, on which the words do matter for identity of logical forms, so that logical forms are sentence types. (For example, Putnam takes such a view, in his discussion of Mates' puzzle in "Synonymy and the Analysis of Belief Sentences,"[1] and Lepore and Ludwig implicitly recognize the distinction in their discussion of examples (4) - (10).)

Now, one would think that on this definition of sameness of logical form, that if there are isomorphic

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[1]Putnam argues that substitution in belief contexts requires identity of logical form, (not, in the context of the discussion, just intensional isomorphism). Consequently, "Nobody doubts that whoever believes that all Greeks are Greeks believes that all Greeks are Greeks" and "Nobody doubts that whoever believes that all Greeks are Greeks believes that all Greeks are Hellenes" can have different truth values because "All Greeks are Greeks" has a different logical form than "All Greeks are Hellenes." This could be accommodated within the spirit of Lepore and Ludwig's general constraint that corresponding proofs are proofs using the same axioms and rules in the same way by adding a clause to the definition to the effect that any given axiom must only be used the same number of times in each proof, thereby distinguishing the logical form of "All Greeks and Greeks" from that of "All Greeks and Hellenes," Although there are questions about just how this condition would be formulated (especially with respect to reflexivization and pronominalization), it does make for a finer grained conception, and at least allows us to reinstate Putnam's account of Mates' puzzle. On the other hand, Lepore and Ludwig might not be terribly upset about losing out on this account of Mates' puzzle, since I don't think they would hold that logical forms are the sorts of things an agent can believe, as one could believe a proposition or sentence. That's not the sort of thing a logical form is; it is rather a property of a sentence.

proofs for S_1 and S_2 in L, as there are for "John left" and "Mary came," and hence that they have same logical form, that this should be no different than saying they have the same syntactic structure. But this is a conclusion that Lepore and Ludwig deny. They allow that there may be sentences S_1 and S_2 that have the same grammatical form, but will have different logical forms because they have different canonical proofs. That is, S_1 and S_2 will have different logical forms just in case their logical forms are determined *non-compositionally*; not as a function of syntactic structure. It is only to the extent that a language is non-compositional that arguably it has non-trivial logical forms.

How would this be? Suppose that S_1 and S_2 have the same syntactic structure. Then the only way they can differ is if they contain different lexical items. Let us say that S_1 and S_2 are the same, except that where S_1 has the word w_1, S_2 has w_2. The axioms of the truth-theory for w_1 and w_2 must be such that what is introduced on their right-hand sides causes the canonical proofs of S_1 and S_2 to differ, to have different numbers of steps. Then, because one proof is more complicated than the other, S_1 and S_2 will have different logical forms, as a reflex of their word-meanings. The difference in logical form thus results from "structure" which is introduced only in the course of semantic interpretation. Hence, the identification by Lepore and Ludwig of logical form with semantic structure; i.e. the structure relevant to the proofs of T-sentences.

It seems then that we have the following. For Lepore and Ludwig, "the relation of sameness of logical form is conceptually basic," where S_1 and S_2 have the same logical form iff they have corresponding canonical proofs of their T-sentences. But if natural languages are compositional, then logical form is the same as grammatical form, and this definition will be equivalent to one based on inspection of the syntactic structures. It is only if natural languages are non-compositional that logical form is distinct from grammatical form, and then of course the definitions will diverge. Evidence for such a divergence would arise from cases in which there are sentences with the same syntactic structure, but with different interpretations, in the sense of different canonical proofs. I take it that this is the point of the Lepore and Ludwig's example of "John is a large man" versus "John is a bald man." For this argument to go through, it must be that pairs of

sentences like these are "minimal pairs." There must be nothing else syntactically different about them other than the different pre-nominal adjectives (i.e. the terminals "large" and "bald"), for if there were, this could give rise to different canonical proofs, and different logical forms, even if the difference were not one connected to the difference in meaning between the sentences. (All that would be necessary would be for one proof to be longer than the other.) That they are a minimal pair is something that is to be established on the basis of independent syntactic evidence. (It would not do to simply observe that they are point-by-point analogous as strings of words, as distinct structures may specify the same linear order of terminals). Given that this has been established, then any semantic difference between the sentences in the pair captured by the interpretative axioms would support an argument for a distinctive notion of logical form.

This sort of semantic difference in the face of grammatical constancy leads Lepore and Ludwig to remark that "Grammatical role often provides a guide to semantic role, but it is an uncertain guide at best . . . it is the semantical role of words which should be our guide. Our aim," they say, "will be to assign different syntactical forms where there are differences in semantic structure." (p. 15) This, however, appears to lead to something paradoxical. The point was to argue that while "John is a large man" and "John is a bald man" "share grammatical form," they have different semantic structure (= logical form); but, because they have different semantic structures, they should be assigned *different* syntactic forms, not the same. But, if they have different syntactic forms, then there is no reason to think in the first place that interpretation is non-compositional; different syntactic structures always have different canonical proofs, and correspondingly different logical forms, but again, logical form will be understood in the trivial sense.

I expect at this point to hear screams of being misquoted, and for good reason, as I have. In the quotation above I left out the words "For the purposes of regimenting sentences for input to a formal semantic theory." According to Lepore and Ludwig, such regimentation is necessary, in part, because "ambiguous natural language sentences lack unique logical forms. In these cases, we must first disambiguate the language before we apply a truth theory to it." There need to be "transformations before applying a truth the-

ory." (p. 12)   With this proviso, we do indeed get a different picture, for now there is a "regimentation" that imposes itself between the language and the truth theory. These regimentations have syntactic structure that may be distinct from their corresponding grammatical forms in *L*. It is the structure of the regimentation which determines proofs of T-sentences, which in turn allows us to say whether sentences of *L* have the same or different logical forms. So, while "John is a large man" and "John is a tall man" have the same grammatical form, they have different regimentations, and hence different canonical proofs, and so to these sentences different logical forms can be attributed. Similarly, "Every man loves some woman" will have two distinct regimentations, and hence is an ambiguous sentence of the language, having two distinct logical forms. Although Lepore and Ludwig tell us that they don't wish to reify logical forms, aren't these regimentations just representations of the logical forms of sentences? After all, their syntactic structure is just determined by semantic structure, the semantic structure which specifies a sentence's logical form. The question of logical form then becomes the question of the status of the syntactic structures called "regimentations."

I believe that Lepore and Ludwig's answer here would be that the regimentations are not representations of logical form of sentences in the object language, but can be used to "give the logical form" of such sentences. This concept they define as follows: "the relation expressed by 'x in L gives the logical form of y in L'" is true of a 4-tuple $\langle x,L,y,L'\rangle$ just in case x in L is the same in logical form as y in L', and x's syntax understood relative to L makes perspicuous the semantic structure of y in L'" (p. 12) Frankly, I'm not quite sure what this means. If L is to be the regimented language relative to L', (and presuming L' ≠ L), then for x and y to have corresponding canonical proofs, x and y must have equally complex structures. But this is a very strong requirement to place on regimentation; intuitively, too strong. (Think of a regimentation of "Every man left" in first-order logic.) Moreover, if there *is* a canonical proof of y in L', why isn't that sufficient to reveal y's logical form; what more is needed? (There will always be a z in L' with a corresponding proof to y in L', if there is a proof of y in the first place; the only possible exception would be idioms.) I must also admit to not understanding what is meant for the syntax of one language to make the semantic structure of

another perspicuous. What could this be other than for one language to formally *represent* the other? But if what is being formally represented is the semantic structure, then isn't this to represent logical form, for as Lepore and Ludwig say, logical form *is* semantic structure. And then aren't we just back to a conception like that of the tradition of the great philosophers mentioned above of a non-trivial, compositional logical form given by translation?

Lepore and Ludwig point out the need not only for regimentation in their approach, but also the need for "adjusting for differences due to differences in the object language." They mention this in connection with "and" and "but," but assuredly the adjustments will have to be considerably more extensive, especially when we have pairs of sentences with the same predicate-argument structure but which are expressed syntactically in different ways. Various examples come readily to mind; for example, active/passive pairs like "John saw Bill" and "Bill was seen by John," or pairs like "It is likely that John will leave" and "John is likely to leave" (raising). It would seem that such pairs have the same logical form; yet it would also seem that given the greater complexity of passives, for example, they would have longer proofs of their T-sentences, even if it is of the same T-sentence as its active counterparts. The issue here is, I believe, quite general whenever we have sentences of different syntactic structure, but which have the same logical form. One could say that this is again a problem of the regimentation, but bear in mind that this is a pervasive and complex problem; its solution, I suspect, would effectively require assuming the entirety of the syntax of the language.

This last remark indicates an approach to the problems of regimentation and adjustment, one that is in vogue among linguists. This is to accept that regimentations are part of the syntactic structure of sentences of the language, part of their grammatical form. The "transformations" to which Lepore and Ludwig allude are to be taken in the technical sense of linguistic theory. On this view, the structure of logical form is determined by the syntactic rules of the language, and the term is synonymous with "input to the truth theory." The project of "giving the logical form" of a sentence is then the project of making more precise what we mean by a natural language; that is, the things that an interpretative truth theory interprets. Within linguistic

theory, the hallmark of this approach is its postulation of a syntactic level of representation, suggestively called *LF*; it is structures at this level which are semantically interpreted. Taking this view, evidence for logical form - that is, representations at LF - will arise from both syntactic *and* semantic sources. In particular, syntax is not merely indicative, and semantics is not preeminent. For instance, that "John is a large man" and "John is a tall man" have different sorts of meanings could be evidence that they have different sorts of syntactic structures, where this difference in syntactic structure would determine a difference in their types of T-sentences. Moreover, on this view it would not necessarily follow that "ambiguous natural language sentences lack unique logical forms," since there may be no such thing as ambiguous (as opposed to under-determined or vague) natural language *sentences*, that is, syntactic structures for which more than one T-sentence can be proven (in an interpretive truth-theory). Certainly, particular strings of concatenated words can be ambiguous. But this is because given strings can correspond to more than one sentence, and it is sentences which are interpreted. For example the string "every man loves some woman," on this view, corresponds to two sentences, which differ in the scope of the quantifiers. (Glossed formally in linguistic terms as a difference in c-command relations at LF.)

It seems to me that the most promising approach we have to the regimentation and adjustment problems is the LF approach just sketched. It has the virtue of putting syntactic theory at our analytic disposal, (as well as placing us under its empirical constraints), for the project of formulating an interpretative truth theory for natural language. It is the approach I adopted in the aforementioned book and elsewhere, and it has been advocated at length by Larson and Segal in their book *Knowledge of Meaning*, for instance. But as the remarks above indicate, it is an approach in which logical form will have the compositional but trivial sense. Insofar as there would be logical form in the non-compositional, non-trivial sense favored by Lepore and Ludwig, it would have to be because there are aspects of semantic structure that are unregimentable; that is, not arguably representable at LF, (although they would have to expressible in the language, given the sort of embedding of the object language in the metalanguage which is assumed).

Now, where does this all leave me? Back, I think, in my vaguely unsatisfied state about logical form, (although not necessarily about the prospects an interpretive truth for natural language), for I keep coming back to logical form as a trivial notion. My problem, however, arises perhaps out of a certain confusion I have been making. What I've been looking for is what Lepore and Ludwig would call not logical form, but *logical syntax*. I suspect that it is something like this latter notion that many people asking what logical form is are looking for, for it is this notion, unlike logical form on Lepore and Ludwig's view, that is connected to isolating out a class of elements as the *logical terms*. Logical syntax, according to Lepore and Ludwig, *is* a syntactic, representational notion, to "be characterized in terms of a pattern of types of terms in a grammatical expression. . . The aim of identifying logical syntax is that of identifying *syntactical* constants in sentences which help to regiment natural language arguments into classes with shared forms that account for validity or invalidity." (p. 19) It is the "recursive syntactical structures of the language" that determines logical syntax; the logical terms are to be identified by their position in logical syntax, with respect to the recursive role they play.

Leaving aside the practical prospects of carrying out this project, it remains that Lepore and Ludwig are concerned with isolating an intuitive notion of constancy or invariance or "topic neutrality" in language. We can now see that they are arguing for two sorts of invariance. One is semantic; it is what logical form captures. The other is syntactic, and it is what logical syntax captures. Lepore and Ludwig's conception of logical terms arises from the latter syntactic conception, which, as they observe, can be contrasted with a semantic conception of logical terms as those elements whose interpretation is invariant with respect to arbitrary permutations of the universe. These two conceptions will to a large extent overlap with respect to the class of terms identified as logical, and Lepore and Ludwig speak pluralistically about the possible need for both notions. That may be defensible, but it is important to recognize that in identifying the logical terms, the two approaches provide very different information about the terms.

To take an example, consider the treatment of quantifiers. In order to capture the generality of

quantifiers in natural language, Lepore and Ludwig suggest a generalization of the familiar satisfaction clauses for the classical quantifiers. Roughly, the strategy is to have for each object language quantifier a corresponding meta-language quantifier over sequences (or assignments). To take an example, we would say that a sequence g satisfies "Most men(x), x are mortal" iff for most g', which differ from g in at most the value of x, g' satisfies "x is a man" and g' satisfies "x is mortal". From the point of view of an interpretative truth theory, I think this is probably the right thing to do; I did it myself (in "Interpreting Logical Form"). But we must be aware that this tells us very little. While it may be sufficient to qualify "most" as a logical term, it leaves the term itself unanalyzed, in that we are left in the dark as to what "most" *means*. With the classical quantifiers, there has always been an answer of sorts; through the reduction to infinite conjunctions or disjunctions of (closed) sentences, we can fall back on an underlying presumption of a theory of reference and the truth-tables for the connectives. But this method will not tell us what it is what it is to be most, or many, or few things. More generally, it will not tell us what it is to be Q-many things, and hence to be a quantifier; to be the sort of term that has a satisfaction clause of the sort above. What is it that gives sense to the T-sentences for quantified sentences?

The only general answer to this question of which I am aware, (aside from listing the terms), is that provided by the theory of generalized quantifiers. This theory defines, for a domain D, a general class of relations between pairs of subsets of D; each generalized (restricted) quantifier q represents some such relation. For example, "some" is the relation between sets X and Y such that they have a nun-null intersection; "most" is the relation between X and Y such that the set of things which are X and Y is larger than the set of things which are X and not Y. Now, if we ask which of the relations definable in the theory are the *quantifiers*, the answer is that they are just those which respect the semantic invariance condition mentioned above; they respect arbitrary permutations of the domain. (This excludes, for example, proper names and focus operators like "only.") The quantifiers can then be further divided into sub-categories in terms of various semantic properties of the relations, and these sub-categories have, it turns out, linguistic significance. Per-

haps most well-known is that the *decreasing* quantifiers (those that are downward entailing) license negative polarity items such as "any" or "ever." Generalized quantifier theory, therefore, not only provides an answer to the question about what are the quantifiers in natural language - they are the terms which express a generalized quantifier- but also affords concepts which allow for the statement of various generalizations about their linguistic properties.

The question that now arises is that, at least as far as the quantifiers is concerned, if we justify the satisfaction clauses in this way, then why do we also need to define the logical terms syntactically; it appears rather that the syntactic conception is parasitic on the semantic conception. It of course may still be valuable to try to isolate a notion of logical syntax - the syntax of expressions containing logical terms so defined. Indeed, I think it is. But we would now trace that back to general syntactic considerations within the language, placing the issue of logical syntax under the general considerations of regimentation and adjustment as part of syntactic theory, as alluded to above.

In saying this, I am not unaware of Lepore and Ludwig's observation that the syntactic and semantic conceptions of logical terms generate different classes of terms, and that there may be arguments that one or the other of the approaches misclassifies certain terms. One case where I think they believe this arguably so is the identity sign, which on the syntactic conception is not a logical term, although there are aspects of its semantics that give it much in common with logical terms. They say:

> The identity sign holds between any object and itself, and the semantic features of the terms which flank it to which it is sensitive are restricted to their being referring terms and bearing the relation of coreference to one another. Sentences in which the identity sign appears, then, although they are not thereby metalinguistic, will be sensitive to relations between terms (or variables) that are abstract semantic relations.

Leaving aside the obvious non sequitur occasioned by the use of the term "sign" in the first line, this comment appears to echo remarks one finds quite commonly in the contemporary literature (cf. Perry, Wettstein) that the analysis of identity statements must refer to the expressions related, although this is not part, if you will, of the proposition expressed. Lepore and Ludwig, however, seem to be proposing an account of identity

statements in the above passage, that is, as far as I know, is completely novel. If I understand what they say, (and I'm not altogether sure that I do), they are claiming that the grammatical identity statements are coextensive with those which are true, and the ungrammatical identity statements are coextensive with those which are false. For if it is a *restriction* placed by the identity sign that the terms flanking it be coreferential, then the only well-formed identity statements are those which satisfy this restriction; hence, any well-formed identity statement must be true. (And any false identity statement, that is, one in which the terms flanking the identity sign do not corefer, does not satisfy the restriction, and hence is ungrammatical.) That the terms which flank the identity sign are coreferential certainly must obtain for an identity statement to be true, and someone who takes "Cicero is Tully" to be true would certainly come to believe that "Cicero" and "Tully" corefer. But I don't see in what sense that there is an "abstract semantic relation" of coreference which must hold independently of the terms flanking the identity sign. If there were, arguably, all true identity statements of the form $\ulcorner a = b \urcorner$ would be analytic, for what information would the identity sign provide not otherwise already expressed by the coreference relation holding between the terms? I presume it is pretty obvious why this account of identity statements is novel, and I'm pretty sure Lepore and Ludwig don't really want to say that all and only grammatical identity statements are true, but then again, I'm far from sure what they do want to say about identity statements.

To summarize, let me say what I think some of the issues here are. (1) I'm unsure, in the end, whether there is an issue about logical form; this strikes me as very much a matter of terminology vis-a-vis the problem of defining the "input" to the truth theory. In large part this is because so much hinges on status of regimentation and adjustment, as this effects what are corresponding proofs, and at this point, these are unanalyzed concepts in the theory; (2) I do think there is an issue lurking about word semantics. Lepore and Ludwig are of the view that differences in types of word-meanings can lead to differences of logical form without any concomitant distinction in syntactic structure, the differences being introduced in the right-hand sides of interpretive axioms. Arguably, this is not right; the alternative is that such differences in word-

meaning do in general have syntactic effects, and if they do, this will be enough to insure, given Lepore and Ludwig's conception, different logical forms. But again, this is a matter of analysis, in the context, I think, of how "regimentation" and "adjustment" are to be specified within an overall understanding of the relation of the syntax and semantic of natural language; (3) With regard to logical syntax, I do think this is a substantive syntactic, representational notion to be isolated in natural language syntax. But I am unsure, however, exactly how the recursiveness concept will be appropriately isolated, as a syntactic concept, short of listing the logical terms. Determining logical syntax is a complex analytic problem, and the machinery needed to characterize it may be both of a syntactic and semantic nature. (4) Finally, we should be mindful that Lepore and Ludwig do us a service, from the perspective of semantics of natural language, of forcing us to clarify with increasing care how syntax and semantics are related, and how this relation gives foundation to why the terms "logic" and "form" seem to be such willing bedfellows.